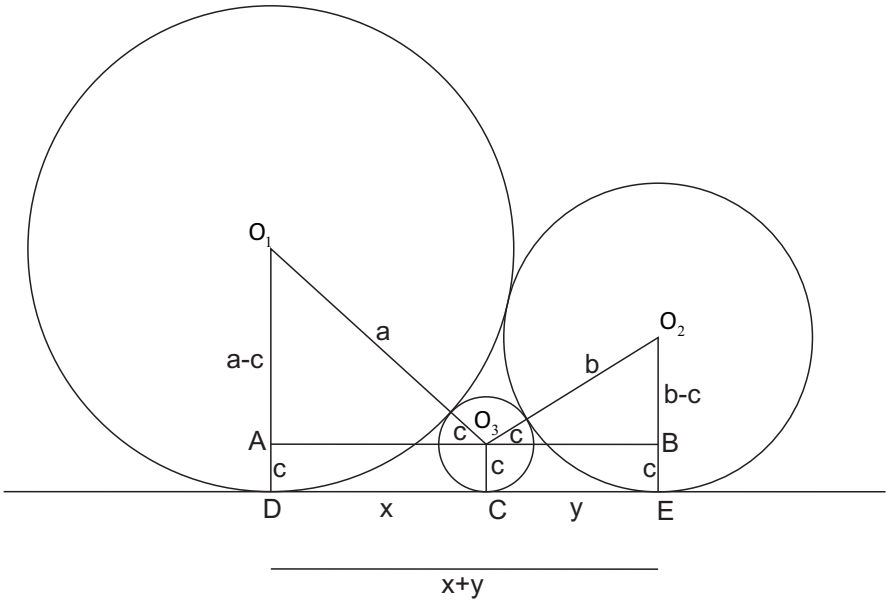
# 1.Aşama

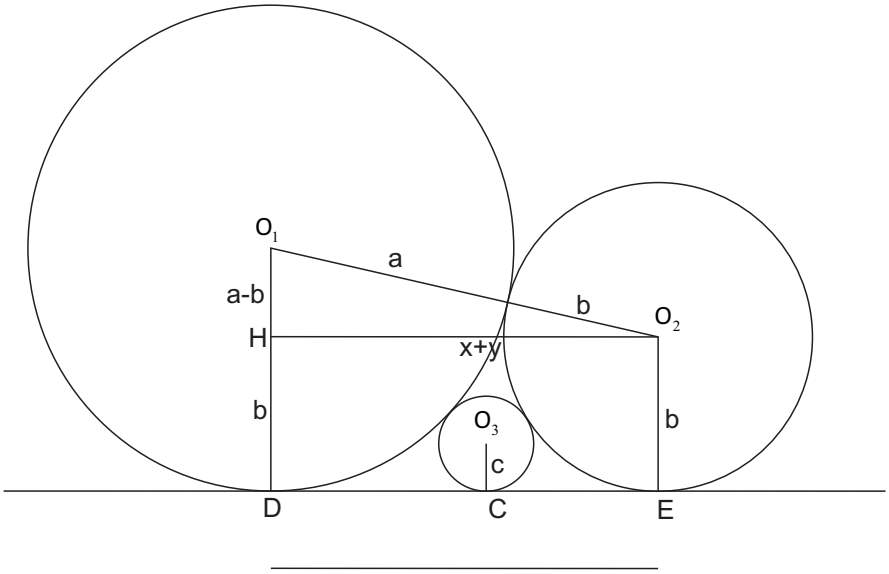



$O_1AO_3$ dik üçgeninde;

$(a+c)^2-(a-c)^2=x^2 \Rightarrow 4ac=x^2$ (I)

$O_2BO_3$ dik üçgeninde;

$(b+c)^2-(b-c)^2=y^2 \Rightarrow 4bc=y^2$ (II)




$O_1HO_2$ dik üçgeninde;

$(a+b)^2-(a-b)^2=(x+y)^2 \Rightarrow 4ab=(x+y)^2$ (III)

Bu 3 denklemden;

$\sqrt{ab}=\sqrt{ac}+\sqrt{bc} \Rightarrow \sqrt{c}=\dfrac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$ elde edilir.

BARIŞ DEMİR

# 2.Aşama

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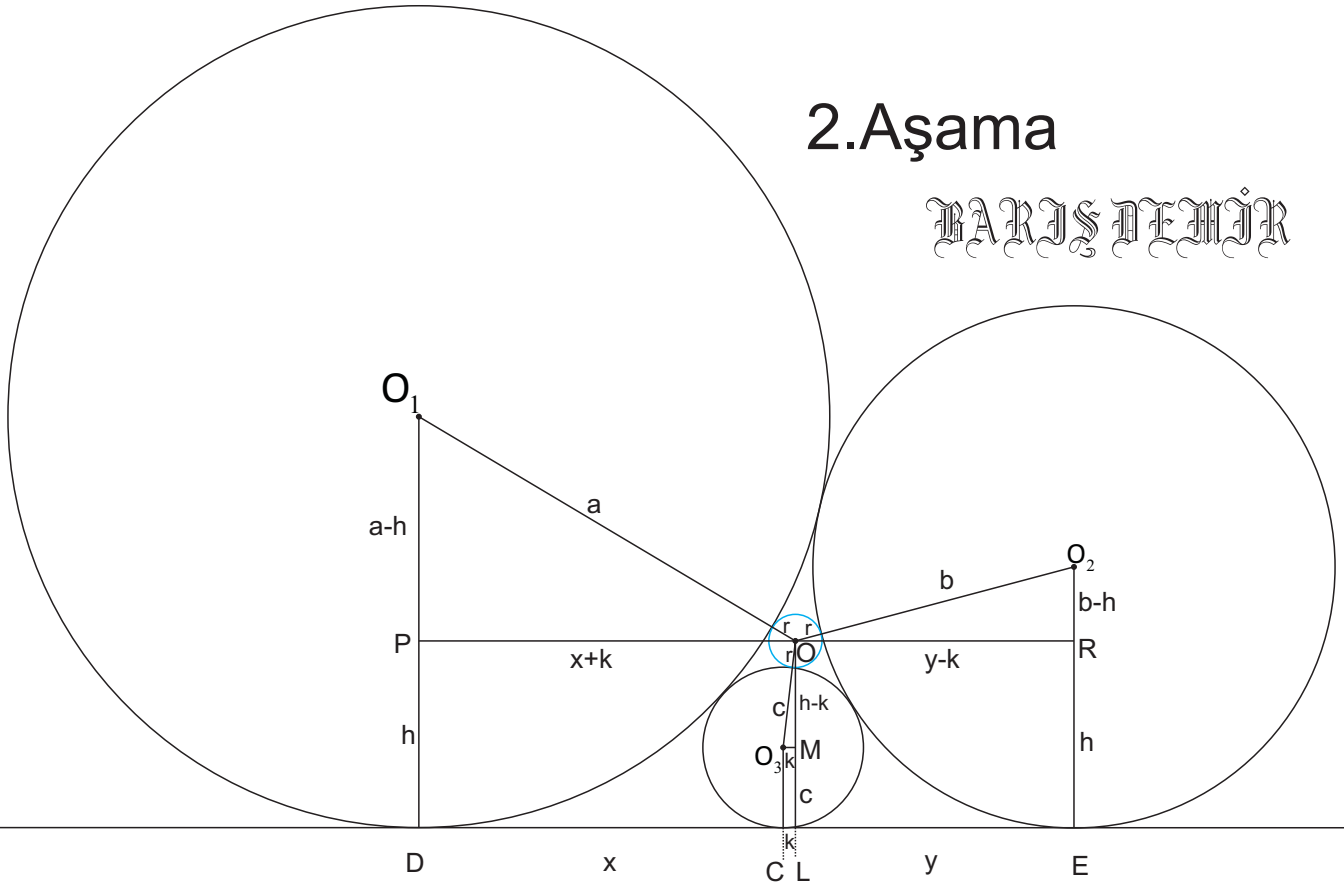


x+y

$|OL|=|RE|=|PD|=h$ , $|CL|=k$ ve O merkezli mavi çemberin yarıçapı r olsun.

$O_1PO$ dik üçgeninden; $(a+r)^2-(a-h)^2=(x+k)^2$ (i) $\Rightarrow$ $(x+k)^2=(r+h).(2a+r-h)$ (d1)

$O_2OR$ dik üçgeninden; $(b+r)^2-(b-h)^2=(y-k)^2$ (ii) $\Rightarrow$ $(y-k)^2=(r+h).(2b+r-h)$ (d2)

$O_3OM$ dik üçgeninden; $(c+r)^2-(h-k)^2=k^2$ (iii) $\Rightarrow$ $(k)^2=(r+h).(2c+r-h)$ (d3) denklemleri elde edilir.

(i) ve (ii) denklemleri taraf tarafa çıkarılırsa; $2(a-b)(r+h)=(x-y+2k).(x+y)$ (d4)

(i) ve (iii) denklemleri taraf tarafa çıkarılırsa; $2(a-c)(r+h)=(x+2k).(x)$ (d5)

(ii) ve (iii) denklemleri taraf tarafa çıkarılırsa; $2(b-c)(r+h)=(y-2k).(y)$ (d6) denklemleri elde edilir.

(d5) ile (d6) denklemleri oranlanırsa; $\frac{a-c}{b-c}=\frac{(x+2k).x}{(y-2k).y}$ $\Rightarrow$ $2k=\frac{y^2(a-c)-x^2(b-c)}{x(b-c)+y(a-c)}$ elde edilir.

Birinci aşamada elde edilen, $x=2\sqrt{ac}$ , $y=2\sqrt{bc}$ ve $\sqrt{c}=\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$ değerleri yerine yazılırsa;

$$2k=\frac{4bc(a-c)-4ac(b-c)}{2\sqrt{ac}(b-c)+2\sqrt{bc}(a-c)}$$

$$\Rightarrow\ k=\frac{\sqrt{c}\left(b(a-c)-a(b-c)\right)}{\sqrt{a}(b-c)+\sqrt{b}(a-c)}=\frac{c\sqrt{c}(a-b)}{\sqrt{a}(b-c)+\sqrt{b}(a-c)}=\frac{\frac{ab}{\left(\sqrt{a}+\sqrt{b}\right)^2}.\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}.(a-b)}{\sqrt{a}\left(b-\frac{ab}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)+\sqrt{b}\left(a-\frac{ab}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}$$

$$=\frac{ab\sqrt{ab}(a-b)}{\left(\sqrt{a}+\sqrt{b}\right)^3}.\frac{1}{\sqrt{a}.\left(\frac{b^2+2b\sqrt{ab}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)+\sqrt{b}.\left(\frac{a^2+2a\sqrt{ab}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}=\frac{ab\sqrt{ab}(a-b)}{\left(\sqrt{a}+\sqrt{b}\right)^3}.\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{b^2\sqrt{a}+2ab\sqrt{b}+a^2\sqrt{b}+2ab\sqrt{a}}=\frac{ab\sqrt{ab}(\sqrt{a}-\sqrt{b})}{b^2\sqrt{a}+2ab\sqrt{b}+a^2\sqrt{b}+2ab\sqrt{a}}$$

$$=\frac{ab\sqrt{ab}(\sqrt{a}-\sqrt{b})}{\sqrt{ab}\left(b\sqrt{b}+2b\sqrt{a}+a\sqrt{a}+2a\sqrt{b}\right)}=\frac{ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}\ \Rightarrow\ \boxed{k=\frac{ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}}$$ elde edilir.

# 2.Aşama devamı

Şimdi

(d1) ile (d4) ü çarpalım: $\frac{2a+r-h}{2(a-b)}=\frac{(x+k)^2}{(x-y+2k)(x+y)} \Rightarrow \boxed{r-h=\frac{2(a-b)(x+k)^2}{(x-y+2k)(x+y)}-2a}$

ayrıca (d4) ten $\boxed{r+h=\frac{(x-y+2k)(x+y)}{2(a-b)}}$ elde edilir.

Bu iki denklem taraf tarafa toplanırsa;

$\boxed{2r=\frac{(x-y+2k)(x+y)}{2(a-b)}+\frac{2(a-b)(x+k)^2}{(x-y+2k)(x+y)}-2a}$ olur. Şimdi; $\frac{2(a-b)}{(x-y+2k)(x+y)}=\Delta$ olsun.

$\Rightarrow 2r=\frac{1}{\Delta}+\Delta.(x+k)^2-2a$ olur.

Öncelikle $x-y+2k=2\sqrt{c}(\sqrt{a}-\sqrt{b})+\frac{2ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}=\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}.(\sqrt{a}-\sqrt{b})+\frac{2ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}$

$=\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}.(\sqrt{a}-\sqrt{b})\left(1+\frac{\sqrt{ab}}{a+b+\sqrt{ab}}\right)=\frac{2\sqrt{ab}(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}+\sqrt{b})(a+b+\sqrt{ab})}=\frac{2\sqrt{ab}(a-b)}{a+b+\sqrt{ab}} \Rightarrow \boxed{x-y+2k=\frac{2\sqrt{ab}(a-b)}{a+b+\sqrt{ab}}}$

$\Rightarrow \Delta=\frac{2(a-b)}{(x-y+2k)(x+y)}=\frac{2(a-b)}{\frac{2\sqrt{ab}(a-b)}{a+b+\sqrt{ab}}.2\sqrt{ab}}=\frac{a+b+\sqrt{ab}}{2ab} \Rightarrow \boxed{\Delta=\frac{a+b+\sqrt{ab}}{2ab}}$ bulunur.

Ayrıca; $x+k=2\sqrt{ac}+\frac{ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}=2\sqrt{a}\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}+\frac{ab(\sqrt{a}-\sqrt{b})}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)}=\frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}\left(2+\frac{\sqrt{b}(\sqrt{a}-\sqrt{b})}{a+b+\sqrt{ab}}\right)$

$=\frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}\left(\frac{2a+b+3\sqrt{ab}}{a+b+\sqrt{ab}}\right)=\frac{a\sqrt{b}\left(2\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(a+b+\sqrt{ab}\right)} \Rightarrow \boxed{x+k=\frac{a\sqrt{b}\left(2\sqrt{a}+\sqrt{b}\right)}{a+b+\sqrt{ab}}}$

Bulunan bu değerler $2r=\frac{1}{\Delta}+\Delta.(x+k)^2-2a$ ifadesinde yerine yazılırsa;

$2r=\frac{1}{\frac{a+b+\sqrt{ab}}{2ab}}+\frac{a+b+\sqrt{ab}}{2ab}.\left(\frac{a\sqrt{b}\left(2\sqrt{a}+\sqrt{b}\right)}{a+b+\sqrt{ab}}\right)^2-2a=\frac{2ab}{a+b+\sqrt{ab}}+\frac{\left(a+b+\sqrt{ab}\right)}{2ab}.\frac{a^2b.\left(2\sqrt{a}+\sqrt{b}\right)^2}{\left(a+b+\sqrt{ab}\right)^2}-2a$

$2r=\frac{2ab}{a+b+\sqrt{ab}}+\frac{a.\left(2\sqrt{a}+\sqrt{b}\right)^2}{2\left(a+b+\sqrt{ab}\right)}-2a=a.\left(\frac{4b+4a+b+4\sqrt{ab}}{2\left(a+b+\sqrt{ab}\right)}-2\right)=a.\left(\frac{4b+4a+b+4\sqrt{ab}-4a-4b-4\sqrt{ab}}{2\left(a+b+\sqrt{ab}\right)}\right)$

$2r=\frac{ab}{2\left(a+b+\sqrt{ab}\right)} \Rightarrow$ nihayet $\boxed{r=\frac{ab}{4\left(a+b+\sqrt{ab}\right)}}$ bulunur.

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