Prove the identity:

$$(\tan(3s) + 4\sin(2s))^2 = 11 - \frac{(\tan(8s) + \tan(3s))\cos(8s)}{\sin(s)\cos(3s)} \tag{1}$$

Proof: The identity (1) is equivalent to the identity (2):

$$11\cos^2(3s) - (\sin(3s) + 4\sin(2s)\cos(3s))^2 = \frac{\sin(11s)}{\sin(s)}, \tag{2}$$

since

$$\tan(8s) + \tan(3s) = \frac{\sin(8s)}{\cos(8s)} + \frac{\sin(3s)}{\cos(3s)} = \frac{\sin(11s)}{\cos(3s)\cos(8s)}.$$

Let $z = e^{is}$, then $\bar{z} = e^{-is}$, $z\bar{z} = 1$, $z^n + \bar{z}^n = 2\cos(ns)$ and $z^n - \bar{z}^n = 2i\sin(ns)$. Then the identity (2) is equivalent to the identity (3):

$$11\left(\frac{z^3 + \bar{z}^3}{2}\right)^2 - \left(\frac{z^3 - \bar{z}^3}{2i} + 4 \cdot \frac{z^2 - \bar{z}^2}{2i} \cdot \frac{z^3 + \bar{z}^3}{2}\right)^2 = \frac{z^{11} - \bar{z}^{11}}{z - \bar{z}} \tag{3}$$

or equivalent to the identity (4):

$$11\left(z^3 + \bar{z}^3\right)^2 + \left((z^3 - \bar{z}^3) + 2(z^2 - \bar{z}^2)(z^3 + \bar{z}^3)\right)^2 = 4 \cdot \frac{z^{11} - \bar{z}^{11}}{z - \bar{z}} \tag{4}$$

The verification of (4) is fairly straightforward although a little tedious.

$$\begin{aligned}
\text{LHS} &= 11(z^3 + \bar{z}^3)^2 + ((z^3 - \bar{z}^3) + 2(z^2 - \bar{z}^2)(z^3 + \bar{z}^3))^2 \\
&= 11(z^6 + 2 + \bar{z}^6) + ((z^3 - \bar{z}^3) + 2(z^5 - \bar{z}^5) - 2(z - \bar{z}))^2 \\
&= 11(z^6 + 2 + \bar{z}^6) + (z^3 - \bar{z}^3)^2 + 4(z^5 - \bar{z}^5)^2 + 4(z - \bar{z})^2 + \\
&\quad 4(z^3 - \bar{z}^3)(z^5 - \bar{z}^5) - 4(z^3 - \bar{z}^3)(z - \bar{z}) - 8(z^5 - \bar{z}^5)(z - \bar{z}) \\
&= 11(z^6 + 2 + \bar{z}^6) + (z^6 - 2 + \bar{z}^6) + 4(z^{10} - 2 + \bar{z}^{10}) + 4(z^2 - 2 + \bar{z}^2) + \\
&\quad 4(z^8 + \bar{z}^8 - z^2 - \bar{z}^2) - 4(z^4 + \bar{z}^4 - z^2 - \bar{z}^2) - 8(z^6 + \bar{z}^6 - z^4 - \bar{z}^4) \\
&= 4(z^{10} + z^8 + z^6 + z^4 + z^2 + 1 + \bar{z}^2 + \bar{z}^4 + \bar{z}^6 + \bar{z}^8 + \bar{z}^{10}) \\
&= 4 \cdot \frac{z^{11} - \bar{z}^{11}}{z - \bar{z}} = \text{RHS}
\end{aligned}$$