

# Weekly Olympiad 2

Day 1



February 18, 2016

## Problems

**Problem 1.** (a) Find all functions  $f: R_+ \rightarrow R_+$  satisfying :

$$f(f(x) - x) = 2x$$

for all  $x > 0$ .

(b) Let  $a, b, c \in R^+$ . Prove the inequality

$$\sum_{cyc} \frac{a^4 + b^4 + a^3 + ab^2}{ab} \geq \frac{8(a+b+c)^2}{a+b+c+3}.$$

**Problem 2.** Let  $ABC$  be a triangle inscribed circle  $(O)$  and orthocenter  $H$ .  $HB, HC$  cut  $(O)$  again at  $E, F$ .  $OE, OF$  cut  $CA, AB$  at  $M, N$ . Construct parallelogram  $ABDC$ . Let  $X, Y, Z$  be reflections of  $H$  through  $MN, ND, DM$ . Prove that circumcircle of triangle  $XYZ$  is tangent to  $(O)$ .

**Problem 3.** In acute triangle  $ABC$ , let  $P$  be a variable point on the minor arc  $BC$ . Let  $AP \cap BC = D$ , and let  $I_1, I_2$  be the incenters of  $\triangle ABP, \triangle ACP$  respectively and let  $J_1, J_2$  be the incenters of  $\triangle BPD, \triangle CPD$  respectively. Let the tangent at  $P$  to the circumcircle of  $\triangle ABC$  meet  $BC$  at  $T$  and let  $I$  be the incenter of  $\triangle ABC$ . Prove that the following are equivalent:

- 1.) Points  $J_1, J_2, T$  are col-linear.
- 2.) Lines  $AI$  and  $I_1I_2$  are perpendicular.

Language : English

Time : 4 hours and 30 minutes

*Each problem is worth 7 points*