Weekly Olympiad 2

Day 2

February 19, 2016

Problems

Problem 4. Find all positive integers $n \geq 2$ such that for all integers i, j that $0 \leq i, j \leq n$, $i + j$ and $\binom{n}{i} + \binom{n}{j}$ have same parity.

Problem 5. Let $ABCDEF$ be convex hexagon inscribed circle (O) with $BC = AF$. AE, BD cut DF, CE at M, N, reps. Circle (K) touches segment ME, MD and touches (O) internally. Circle (L) touches segment NE, ND and touches (O) internally. Prove that there is a common tangent of $(K), (L)$ that is parallel to AB.

Problem 6. At each vertex of a regular 1997-gon is written a positive integer. One may add 2 to any of the numbers and subtract 1 from the numbers k away from it in either direction, for some $k \in \{1, 2, ..., 1998\}$; the number k is then written on a blackboard. After some number of operations, the original numbers reappear at all of the vertices. Prove that at this time, the sum of the squares of the numbers written on the blackboard is divisible by 1997.

Language : English *Time : 4 hours and 30 minutes*

Each problem is worth 7 points