Lemmas in Geometry

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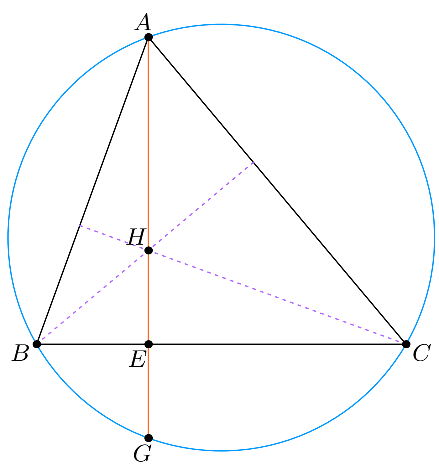
February 10th 2016

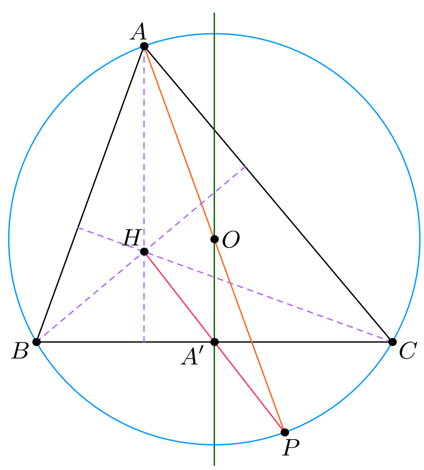
1 Lemmas in Euclidean Geometry

In this Article, we shall explore a few lemmas in Euclidean Geometry. This article will be updated every week with more lemmas as and when I gather more – so keep checking! And have fun seeing problems dissolve as you apply this collection of lemmas in solving.

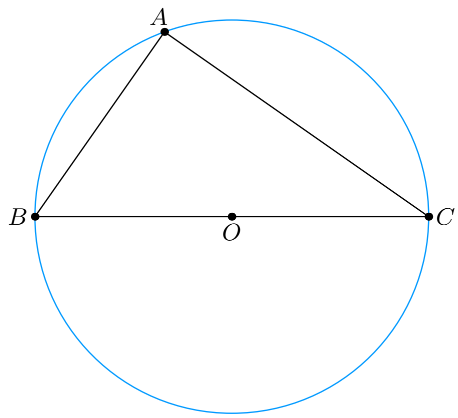
Lemma 1. Reflection of the Orthocentre about any side lies on the Circumcircle of the triangle, i.e. $HE = EG$.



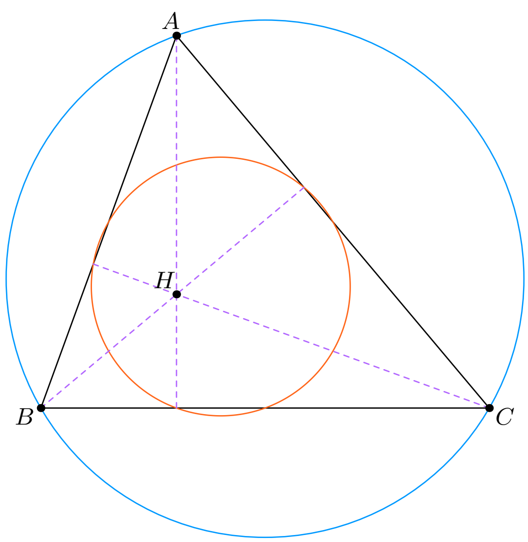
Lemma 2. If AP is a circumdiameter, then PH and BC bisect each other. If A' is the foot of perpendicular from O to BC, then AH = 2OA'



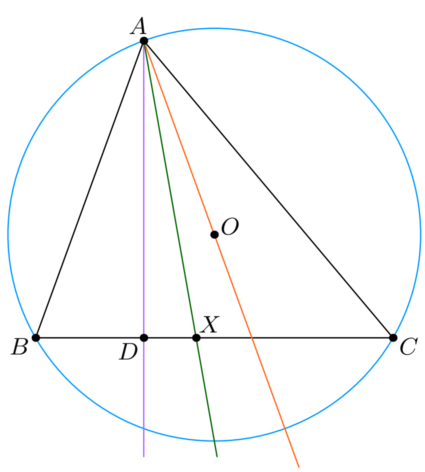
Lemma 3. The midpoint of the hypotenuse of a right angled triangle is its circumcentre. Can be used in various ways.



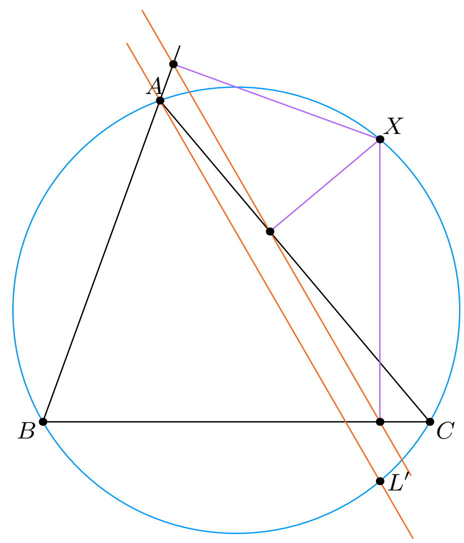
Lemma 4. Nine point circle's radius is half the circumradius.



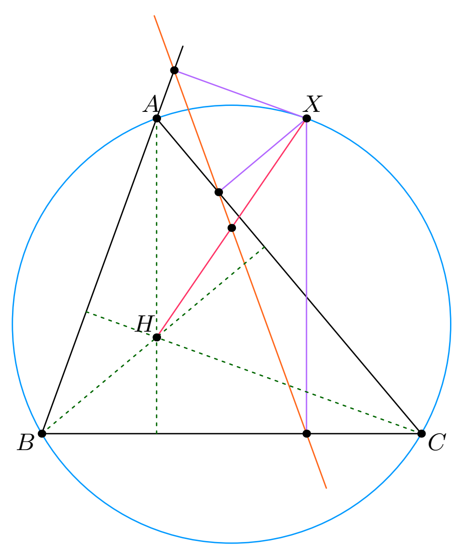
Lemma 5. AD and AO are reflections of each other in the angle bisector, AX.



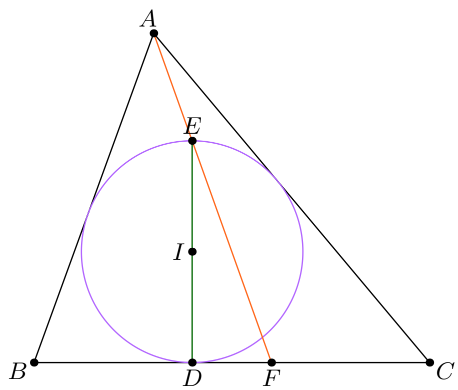
Lemma 6. If X is a point on the circumcircle of ABC, and the perpendicular to BC from X intersects the circumcircle at L', then AL' is parallel to the Simson line.



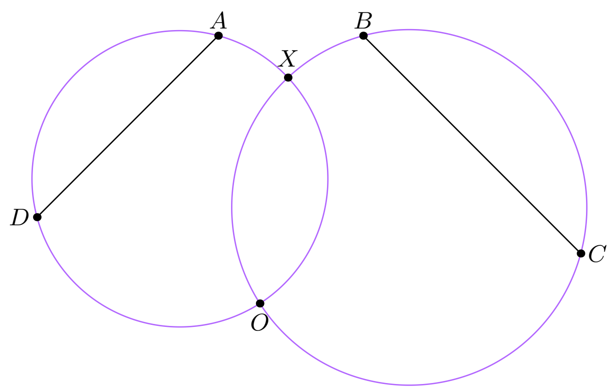
Lemma 7. The Simson line wrt X bisects the line joining X and the Orthocentre.



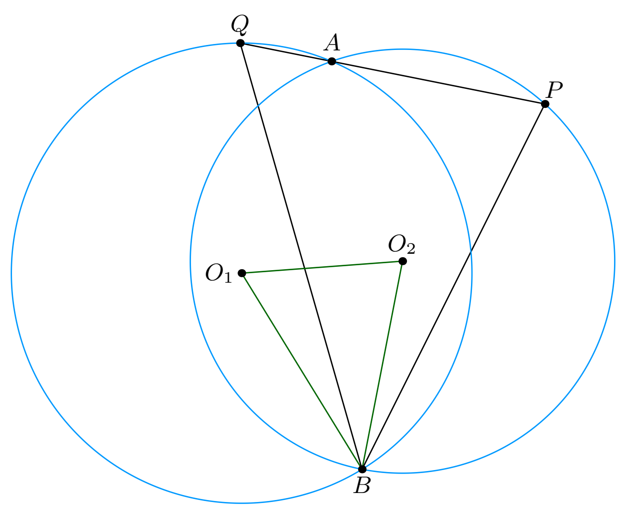
Lemma 8. Let incircle of ABC touch side BC at D, and let DE be a diameter of the circle. If line AE meets BC at F, then BD=CF.



Lemma 9. Let AD and BC be two segments, and let AC and BD meet at X.Let the circumcircles of ADX and BCX meet again at O. Then O is the centre of spiral similarity that carries AD to BC.



Lemma 10. If on chord QP, A is chosen, and B is any point not on QP, then if O_1 is the circumcentre of AQB and O_2 is the circumcentre of PAB, then there is spiral homothety between BO_1O_2, BQP and B is centre of spiral similarity.



Lemma 11. Suppose that the circles ω_1 and ω_2 intersect at distinct points A and B. Let CD be any chord on ω_1, and let E and F be the second intersections of the lines CA and BD respectively, with ω_2. Then EF is

parallel to CD.